FUZZY MODEL FOR DETECTION OF FRAUDULENT FINANCIAL STATEMENTS: A CASE STUDY OF LITHUANIAN MICRO AND SMALL ENTERPRISES

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ABSTRACT

90 per cent of enterprises in the European Union (EU), including Lithuania, are small enterprises that prepare the abridged financial statements. Verifying the fairness of these reports for stakeholders is challenged due to the lack of data. The aim of this research is to develop a novel model based on fuzzy logic for detecting fraudulent financial statements in micro and small enterprises by using financial ratios suitable for abridged financial statements. The results have shown that the developed fuzzy model enables estimation of the level of fraud in each individual element of accounting. Identifying each fraudulent accounting element allows us to gain insights into the areas where the enterprise has committed fraud. The proposed model has been designed to help small businesses reduce the risk, but it may also be used by public authorities as a tool for achieving greater business transparency.

KEY WORDS

accounting, financial statement, fraud, fuzzy

JEL CODES

G3, M41

1 INTRODUCTION

The users of financial statements in the EU face the challenge when performing the analysis of financial statements between enterprises of different sizes and between enterprises operating in different EU countries. It is not enough to be well acquainted with the Accounting Directive 2013/34/EU as certain differences in the financial statement information depend on the national legislation (Hýblová, 2019). There are two implications: the diversity of financial statement structures and the lack of an available financial statement analysis tool for micro and small enterprises that would account for the specifics of national legislation. Therefore,

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small businesses need a reliable innovative tool for detection of fraudulent financial statements. The present paper is the continuation of the research under the previous technical feasibility study "Feasibility study on development of an innovative tool for detection of falsification of financial statements" (2021, No. 01.2.1-MITA-T-851-02-0171).

Various studies have been conducted by researchers in this area. The listed companies that belong to the group of large undertakings and prepare the detailed version of financial statements are investigated in the studies related to detection of fraudulent financial statements (Rostamy-Malkhalifeh et al., 2021; Hakami et al., 2020; Aghghaleh et al., 2016; Chen et al., 2014; and others). Researchers often face barriers when collecting data on micro or small enterprises. As a result, the studies largely use the publicly available data on listed companies. Hence, the present research focuses on the data of financial statements of micro and small enterprises and aims at designing a model for detection of fraudulent financial statements that would be applicable to the abridged financial statement versions.

The aim of the research is to develop a novel model based on fuzzy logic for detecting fraudulent financial statements in micro and small enterprises by using financial ratios and models suitable for abridged financial statements. One of the added values of this empirical research is that the presented set of financial ratios and models is suitable for analysing abridged financial statements, as previous studies paid little attention to this and focused on the financial statements of large, listed companies. Another unique aspect of this empirical research is that cases of fraudulent financial statements can be identified from abridged financial statements without the need for additional non-publicly available information. Lastly, the designed fuzzy model for detection of fraudulent financial statements is easily applicable and useful in practice, as well as for conducting further scientific research.

The next section (Section 2) of the paper provides the overview of the selected financial ratios and models used for detection of financial fraud and innovative artificial intelligence (AI) research methods used for detection of fraudulent financial statements. Section 3 presents the research methodology, the proposed fuzzy model for detection of fraudulent financial statements. The results of the empirical analysis and discussion are presented in Section 4. The conclusions are presented in the last section.

2 LITERATURE REVIEW

2.1 Financial Ratios and Models Used for Detection of Fraudulent Financial Statements

Information provided in the financial statements enables fast assessment of the financial situation at an enterprise for decision-making purposes. Financial statements are aggregated in the financial accounting data that reveal no more than 50 per cent of effective information that is necessary when making financial decisions (Besusparienė et al., 2018). Financial information is nevertheless necessary in the managerial (Besusparienė et al., 2018), investment, and financial decision-making (Shakespeare, 2020). For the analysis and interpretation of financial statements, various financial ratios or model are commonly used for the cases discussed before.

Since 2013, the European Union (EU) has merged and replaced the earlier accounting directives with the new Accounting Directive 2013/34/EU (Directive 2013/34/EU of the European Parliament ..., 2013). The major change is the diversification of the sets of financial statements according to the size of enterprises and simplifications applicable to micro and small undertakings (Deac, 2014). The Accounting Directive 2013/34/EU (Directive 2013/34/EU of the European Parliament ..., 2013) establishes that the Member States may permit small undertakings to prepare abridged

Tab. 1: Financial ratios used for detection of financial fraud

Financial ratio	Ratio threshold: No fraud	Ratio threshold: Fraud	Applicability to financial statements of small enterprises
Days' sales in receivables index (DSRI)	DSRI $\leq$ 1.030 DSRI $\leq$ 1.031	DSRI $\geq$ 1.460 1 $\leq$ DSRI $\leq$ 1.5 DSRI $\geq$ 1.465	YES
Gross margin index (GMI)	GMI $\leq$ 1.041	GMI $\geq$ 1.190 GMI $\geq$ 1.193 1 $\leq$ GMI $\leq$ 1.2	YES
Sales growth index (SGI)	SGI $\leq$ 1.134 SGI $\leq$ 1.1	SGI $\geq$ 1.610 SGI $\geq$ 1.6 SGI $\geq$ 1.607	YES
Sales, general and administrative expenses index (SGAI)	SGAI $\leq$ 1.001 SGAI $\leq$ 1.054	SGAI $\geq$ 1.041	YES
Net profit and gross profit ratio (GP)	GP $\geq$ 0.362	GP $\leq$ 0.086	YES
Asset quality index (AQI)	AQI $\leq$ 1.040 AQI $\leq$ 1.041	1 $\leq$ AQI $\leq$ 1.25 AQI $\geq$ 1.254 AQI $>$ 1	NO
Deprecation index (DEPI)	DEPI $\leq$ 1.001	DEPI $\geq$ 1.077 DEPI $>$ 1	NO
Total accruals to total assets index (TATA)	TATA $\leq$ 0.018 TATA $\leq$ 0.016	TATA $\geq$ 0.031	NO
Leverage index (LEVI)	LEVI $\leq$ 1.037	LEVI $\geq$ 1.111 LEVI $\geq$ 1	YES
Return on current assets (ROCA)	ROCA $\geq$ 0.299	ROCA $\leq$ 0.057	YES
Stable funding ratio (SFR)	SFR $\geq$ 0.586	SFR $\leq$ 0.453	YES
Asset turnover (ATO)	ATO $\leq$ 1.54	ATO $\geq$ 2.243	YES
Current assets to total assets ratio (CATA)	CATA $\leq$ 0.473	CATA $\geq$ 0.667	YES
Inventory to total assets ratio (ITA)	ITA $\leq$ 0.190	ITA $\geq$ 0.321	YES
Cash to total assets ratio (CTA)	CTA $\leq$ 0.035	CTA $\geq$ 0.091	YES

Source: Wells (2001); Kanapickienė and Grundienė (2014); Mamo and Shehu (2015); MacCarthy (2017); Bhavani and Amponsah (2017)

balance sheets and profit (loss) statements without any obligation to prepare other statements (e.g., cash flow statement or statement of change in equity). The differences lie not only in the sizes of enterprises, but also in the national legal frameworks (Hýblová, 2019).

Certain financial ratios or models are impossible to calculate due to the lack of detailed data in the abridged version balance sheets or of other information provided in other statements (e.g., changes in equity or cash flows statements), which are not prepared in the case of small enterprises. Following the analysis of previous studies, the financial ratios potentially signalling a financial statement fraud were identified. Possible calculation of these financial ratios based on the financial statements of small enterprises is provided below (see Tab. 1).

The literature analysis has shown that, in case small enterprises choose to prepare an abridged balance sheet, the asset quality index (AQI) cannot be calculated as the tangible fixed assets are presented in the aggregate amount, while the data on the amounts of real estate are not available. The depreciation index (DEPI) and total accruals to total assets index (TATA) also cannot be calculated. A balance sheet or profit (loss) statement does not provide information about depreciation in the abridged financial statements. Other financial ratios presented in Tab. 1 can be calculated based on the data in the financial statements of small enterprises.

Tab. 2: Financial models for detection of financial fraud

Financial ratio	Ratio threshold: No fraud	Ratio threshold: Fraud	Applicability to financial statements of small enterprises
Altman model Z score (Mavengere, 2015)	$Z > 2.67$	$Z < 1.81$	YES
Modified Altman model Z score (Bhavani and Amponsah, 2017; Mavengere, 2015)	$Z > 2.67$	$Z < 1.81$	YES
Modified Altman model Z score for non-manufacturing enterprises (Mavengere, 2015)	$Z > 2.6$	$Z < 1.1$	YES
Modified Altman model Z score for developing markets (Al Zaabi, 2011)	$Z > 2.6$	$Z < 1.1$	YES
Beneish model M score (Mamo and Shehu, 2015; MacCarthy, 2017)	$M < -2.22$	$M > -2.22$	NO
Beneish model M score (Halilbegovic et al., 2020)	$M < -1.78$	$M > -1.78$	NO
Linear regression model Y score for pharmacy and automotive enterprises (Taherinia et al., 2019)	$Y < 0.05$	$Y > 0.05$	NO
Logistic regression model P score (Kanapickienė and Grundienė, 2015)	$P < 0.5$	$P > 0.5$	YES
Dechow model F score (Aghghaleh et al., 2016; Hakami et al., 2020)	$F < 1$	$F > 1$	NO
CFEBT model (Drábková, 2016)	$CFEBT > 0.05$ $CFEBT > 0.1$	$CFEBT < 0.05$	NO

Difference between the thresholds of financial ratios has been noticed, and certain researchers leave a grey area (MacCarthy, 2017; Kanapickienė and Grundienė, 2014) when the financial ratios are between fraud and no-fraud. The researchers point out that the values of financial ratios may be affected not only by fraud, but also by legitimate extraordinary operations. For example, the DSRI rate may increase due to changes in the credit policy (Wells, 2001). Therefore, the researchers recommend setting a grey area as a compromise between fraud and no-fraud in the respective situations (Karas and Režňáková, 2020).

In detection of fraud in financial statements, the researchers often integrate financial ratios into the designed financial models for detection of financial fraud. The researchers usually use the Altman model (Aghghaleh et al., 2016; MacCarthy, 2017; Bhavani and Amponsah, 2017; Karas and Režňáková, 2020; Georgiev and Petrova, 2020) and Beneish model (Mamo and Shehu, 2015; Drábková, 2016; MacCarthy, 2017; Bhavani and Amponsah, 2017; Halilbegovic et al., 2020) to disclose fraud in financial statements, while other researchers adapt or modify these models by country or company sectors (Georgiev and Petrova, 2020). The researchers (Aghghaleh et al., 2016; Hakami et al., 2020) also propose applying the Dechow model, which differs from other models in the higher level of probability of fraud in financial statements. Other researchers (Kanapickienė and Grundienė, 2015; Drábková, 2016; Taherinia and Talebi, 2019) have been exploring new models. This demonstrates the diversity of financial models for detection of financial fraud. The score threshold of the models mentioned above is identified in Tab. 2.

However, due to the abridged version of the balance sheet and the lack of additional information result, it is impossible to calculate the majority of models. Therefore, only the Altman Z-score model and logistic regression P-score model have been identified as appropriate for detection of fraudulent financial statements in the case of small enterprises.

Tab. 3: Relationship between elements of accounting and financial ratios and models

	Effect of accounting elements to financial ratios and models					
Financial ratios and models	**Revenue**	**Expenses**	**Profit**	**Assets**	**Equity**	**Liability**
DSRI	+			+		
GMI	+	+				
SGI	+					
SGAI	+	+				
GP			+		+	
LEVI				+		+
ROCA			+	+	+	
SFR				+	+	+
ATO	+			+		
CATA				+		
ITA				+		
CTA				+		
P-score	+			+		+
Z-score	+		+	+	+	+

In summary, twelve financial ratios and two financial models were identified in various empirical studies for detection of fraudulent financial statements applicable to the cases of small enterprises. Depending on the financial data needed for the calculation, the ratios and models have been assigned to the main elements of accounting (see Tab. 3).

Tab. 4 presents the examples of sample sizes used in various financial research works. It can be noted that large data sets are required for application of neural networks. On the other hand, the neuro-fuzzy may use smaller data sets.

The classification presented in Tab. 3 is used in subsequent sections of the present research.

2.2 Innovative Research Methods for Detection of Fraudulent Financial Statements

The early studies of financial fraud were based on the traditional linear financial models presented in Section 2.1. However, there are diverse financial models and ratios. In certain cases, these financial models and ratios may complement each other, while in other cases, they show conflicting results. Therefore, in the recent years, researchers have been employing various AI techniques for fraudulent financial statement investigation. These techniques are used for exploring solutions to complex problems that often require sophisticated systems in order to evaluate the accumulated results based on past data. The key issue was to identify how the respective researchers use various AI techniques to solve financial problems.

Following the systematic literature review, the AI methods used for detection of fraudulent financial statements were identified, such as fuzzy logic (Arora and Saini, 2013; Maltoudoglou et al., 2015; Antonelli et al., 2016; Omar et al., 2017; Nawrocki, 2018; Čičak and Vašiček, 2019; Tiwari et al., 2020), neural networks (Antonelli et al., 2016; Omar et al., 2017; Tiwari et al., 2020), neuro-fuzzy (Arora and Saini, 2013; Antonelli et al., 2016; Rostamy-Malkhalifeh et al., 2021) and decision trees (Arora and Saini, 2013; Chen et al., 2014; Lin et al., 2015; Antonelli et al., 2016; Omar et al., 2017; Zhang, 2020). Hence, previous empirical studies on these AI methods were analysed for finding an appropriate method to us.

Fuzzy systems may be applied well to financial modelling, and they are based on fuzzy set theory and fuzzy multivalued logic (Tiwari et al., 2020). The advantage of application of the fuzzy logic in financial research is that the classical bivalent logic is abandoned and an

Tab. 4: Examples of studies applying neural networks

Type of study	Examples of samples sizes
Neural networks	The sample consists of 550 company data, of which 440 company data are used for neural network training, 110 company data are used for model testing (Omar et al., 2017).
	Data of listed companies on the Bombay Stock Exchange (BSE) from 07/02/2012 to 17/02/2016, of which 70 per cent are used for neural network training and 30 per cent for testing (Tiwari et al., 2020).
	The study sample size was 129 cases of companies' fraud between 1998 and 2010 and 447 companies without fraud according to the established criteria, and 69 companies as controls sample. 70 per cent of fraudulent and non-fraudulent companies were randomly selected and 30 per cent the data was selected for training the data set (Zhang, 2020).
Neuro-fuzzy network	The sample consisted of financial data of 10 listed companies for 2010–2018, of which 80 companies used data for training and 10 companies used data for testing (Rostamy-Malkhalifeh et al., 2021).
	Banking bankruptcies study used 2012 data for training and testing the data – 480 (Arora and Saini, 2013).

alternative way of thinking is chosen. This enables the researchers to model complex systems and use the existing knowledge and experience (Čičak and Vašiček, 2019). In summary, various empirical studies (Huda et al., 2015; Nawrocki, 2018; Čičak and Vašiček, 2019) apply a logical scheme to the financial research based on fuzzy logic. When deciding on the software to be used for fuzzy logic application, certain researchers (Maltoudoglou et al., 2015; Čičak and Vašiček, 2019; Rostamy-Malkhalifeh et al., 2021) tend to opt for MATLAB™ software as it contains several tools for applying of fuzzy logic (Čičak and Vašiček, 2019). In MATLAB™, two fuzzy logic models – Sugeno or Mamdani – may be selected. Compared to the Mamdani method, the Sugeno method is mathematically more efficient in model construction and is applied more frequently to the optimization problems. The Mamdani method seems to be more appropriate and flexible when used for the intuitive models based on human reasoning (Čičak and Vašiček, 2019). Therefore, the Mamdani method is preferred in financial research (Nawrocki, 2018; Čičak and Vašiček, 2019).

In practice, the fuzzy models usually operate with the IF-THEN-type rules when the input and output variables are specified (Nawrocki, 2018; Čičak and Vašiček, 2019). The output values are based on possible combinations of these weighted rules (Čičak and Vašiček, 2019). Huda et al. (2015) propose using the linguistic values for the fraudulence variable by applying a broad scale of the level: no fraud (weight 0.01–0.25), between no fraud and fraud (weight 0.26–0.40), fraud (weight 0.41–0.60), real fraud (weight 0.61–0.75), very real fraud (weight 0.76–1.00).

The other two widely used methods in the financial research are the neural networks and neuro-fuzzy (Arora and Saini, 2013; Omar et al., 2017; Tiwari et al., 2020; Zhang, 2020; Rostamy-Malkhalifeh et al., 2021). Neural networks differ from other research methods in that they are trained to perform certain tasks according to the sample data sets (Omar et al., 2017). The artificial neural networks constitute the input and output layers and between them we may assign one or more hidden layers. These layers include given number of neurons and their interconnections. The neurons are working in parallel thus ensuring more of computational power, flexibility, simplicity and efficiency (Tiwari et al., 2020; Rostamy-Malkhalifeh et al., 2021). In neuro-fuzzy models, in turn, the initial fuzzy models are fine-tuned with neural networks (Arora and Saini, 2013; Antonelli et al., 2016; Rostamy-Malkhalifeh et al., 2021).

The results of empirical research reveal the advantages and disadvantages of comparing the application of fuzzy logic, neural network or neuro-fuzzy network. Tiwari et al. (2020) confirm that the both methods – neural network and fuzzy logic – provide similar results, but the neural networks have performed better in the studies with stock prices. Omar et al. (2017) observed more reliable results provided by neural networks in the case of the fraud financial statement study. Nonetheless, they faced limitations in data sampling size and highlighted the need to use larger data sets for neural network learning.

In the analysis of the fraudulent financial statements, decision trees may also be applied (Chen et al., 2014; Lin et al., 2015; Antonelli et al., 2016; Omar et al., 2017; Zhang, 2020). This method is used for classification or discrimination of the financial variable values. The results generally indicate a high degree of accuracy when company's fraudulent financial statements are assessed by using the available public financial data. However, the results may be different for small enterprises because their financial statements are not publicly available, and the research in relation thereof is insufficient (Omar et al., 2017). A decision tree is constructed by using certain classifying or discriminating elements for the variables (Lin et al., 2015) and then optimal associations between these classes are created (Antonelli et al., 2016). In this context, if-then classification rules are also created (Antonelli et al., 2016; Zhang, 2020) and, finally, the decision tree with the branches is obtained (Lin et al., 2015). The results concerning financial frauds reveal that the reliability of the decision tree is about 90 per cent, same as using the neural networks (Zhang, 2020).

Hence, the fuzzy logic method seems applicable to the problem setting in the context of the present research due to the linguistic variables and simple models. In addition, other methods such as the decision trees, may be applied.

3 METHODOLOGY AND DATA

3.1 Lithuania Case Study and Sampling Characteristic

It is important to mention that micro and small enterprises prevail on the EU market. They are also more vulnerable than medium-sized or large enterprises. Micro and small enterprises together account for about 99 per cent of the total business in the EU (Deac, 2014; European Commission, 2021). According to Eurostat, micro and small enterprises accounted for 98.8 per cent of all enterprises in Lithuania in 2019, same as in 2017 and 2018. According to the statistics, small businesses often failed. According to the data by Eurostat, the number of enterprise deaths was estimated at about 50k enterprises in 2018 in Lithuania. About 99 per cent of deaths among the small enterprises were micro enterprises (with less than four employees). According to the data by Eurostat for 2010–2017, an average of 16 per cent of the active enterprises per year were terminated the Lithuania. Since 2018, the death rate of enterprises started to increase. In 2018, the death rate was almost 18 per cent, and in 2019, more than 22 per cent.

According to the Lithuanian study (AVNT, 2018) based on the analysis of bankruptcy reports of the enterprises, the main reasons for bankruptcy identified were the following: excessive debts accumulated and excessive risks assumed (about 30 per cent); lack of working capital and credit problems (about 22 per cent); loss of market or partners (about 10 per cent); poor corporate governance (about 7 per cent). One of the reasons for debt growth or risk mismanagement may be the fraudulent financial statements.

The Authority of Audit, Accounting, Property Valuation and Insolvency Management (AVNT, 2020) reported the trend of decrease in the enterprise bankruptcy proceedings by about 23 to 30 per cent in 2019–2020. However, the COVID-19 virus, which has disrupted the normal life of society, the lockdown, and financial support measures for the business have influenced the decrease in the number of bankruptcy proceedings (AVNT, 2021).

Court cases on fraudulent accounting reveal the existing problems in Lithuanian enterprises caused by the financial data fraud. The contents of the court cases have revealed that financial fraud occurs when enterprises fail to record the revenue received (Plungės ..., 2020), forge the payment documents (Klaipėdos ..., 2020), and commit other fraud on fixed assets, current assets, cash, or equity (Kauno apylinkės ..., 2021). The court cases have shown that the enterprises submitted fraudulent financial statements to the Register of Legal Entities and other parties (Kauno apygardos teismo teisėja, 2021; Kauno apygardos teismo Civilinių ..., 2021). Fraud in accounting data and financial statements leads not only the bankruptcy of the enterprises themselves, but also to the financial difficulties of the cooperating enterprises.

According to the data by Statistics Lithuania (the Lithuanian Department of Statistics) (eliminating the public sector), there were 101k enterprises with less than 250 employees at the beginning of 2019. For the research to have a 95 per cent confidence interval and meet the 0.06 error requirement, the sample of 383 companies had to be reached. The data were collected during preparation of the previous technical feasibility study "Feasibility study on development of an innovative tool for detection of falsification of financial statements" (2021, No. 01.2.1-MITA-T-851-02-0171) . Data of financial statements of 60 micro or small enterprises for the period 2017–2019 were collected. The error requirement for the research was 0.13 with 95 per cent confidence interval. Due to the variety of data, the financial statements of enterprises were collected by different life cycle; therefore, some of the sampled enterprises did not prepare any statements for each subsequent year in the period 2017–2019.

Of the sampled enterprises, only 11 enterprises prepared financial statements as micro enterprises (the abridged balance sheet and abridged profit (loss) statement by nature). In several cases, certain enterprises prepared different financial statements in different years of the respective period. The research was therefore limited to the version of financial statements intended for small enterprises. The data of the total of 129 financial statements (abridged balance sheet and profit (loss) statement by function) were collected: 33 enterprises in 2017, 46 enterprises in 2018, and 50 enterprises in 2019. After the review of the data of financial statements and selected financial ratios and models in Section 1, certain enterprises had to be excluded. In some cases, the SGAI financial ratio of certain enterprises could not be calculated as no sales and administrative expenses were provided in the profit (loss) statements. New enterprises had not earned sales revenue yet, and it was impossible to calculate the financial ratios of DSRI, GMI and SGI. Certain financial ratios required two years of data; therefore, the enterprises operating in the first year had to be eliminated. For certain enterprises, the logistic regression model by Kanapickienė and Grundienė (2015) could not be calculated as the enterprises did not have any fixed assets.

Following assessment of all the limitations of the present research, the data of 107 financial statements were used: data of 26 enterprises in 2017, 37 enterprises in 2018, 44 enterprises in 2019 (see Tab. 5). Due to the data protection regulation, depersonalised company data were obtained. It was therefore impossible to assess the companies by number of employees or by sector.

Tab. 5: Revenue and asset values in the financial statements used in this research ($N = 107$)

Year	**2017**	**2018**	**2019**
Enterprises (N)	26	37	44
Revenue, Euro			
>700k	50.0%	56.8%	54.5%
700k–8m	50.0%	40.5%	43.2%
8–40m	0.0%	2.7%	2.3%
Assets, Euro			
>350k	26.9%	37.8%	38.6%
350k–4m	69.2%	56.8%	56.8%
4m–20m	3.8%	2.7%	2.3%

The Accounting Directive 2013/34/EU (Directive 2013/34/EU of the European Parliament ..., 2013) of the EU provides for three criteria for assessment of the size of enterprises (revenue, assets, employees number), two of which shall be met within two years. The analysis of the mentioned criteria showed that the enterprises divided evenly into micro and small enterprises according to the revenue criterion. However, the analysis of assets showed that the majority of the enterprises were small enterprises according to the asset criterion. Nevertheless, without any data on employees, it was impossible to make a definitive conclusion as to what share of enterprises were micro or small enterprises.

3.2 Research Methodology

The methodology of the present research was based on the financial ratios and models discussed in Section 2.1 as well as the use of the abridged financial statements intended for small enterprises. The research methods were discussed in Section 2.2. Fig. 1 presents the stages of the research.

First, as presented in Section 2.1, 12 financial ratios and two financial models suitable for the abridged financial statements of micro and small enterprises, were selected. Second, as presented in Tab. 3, the financial ratios and models were grouped into six groups by relationship between the elements of accounting and financial ratios and models. The six groups were transformed into six input variables. Third, the financial ratios and model were calculated with Microsoft Excel™. The results accounting for the thresholds of financial ratios and models were encoded (see Tab. 6) according to the thresholds in Section 2.1. The minimum and maximum values of the thresholds were encoded as no fraud (0) and fraud (1), respectively, and intermediate values, the so-called the grey area, were encoded as possible fraud (0.5).

Then, the average fraud rate was derived from the selected enterprises in each group of the accounting elements by these thresholds (see Tab. 6). The average fraud results of each group by elements of accounting (see Tab. 3) were the values of the six input variables presented in step 2 of our research scheme (see Fig. 1).

As indicated by the analysis in Section 2.2, the neural networks could not be applied in the present research due to the small data set. Therefore, the fuzzy logic was chosen. Stage 4 of the research methodology was dedicated to the design of the fuzzy model for detection of fraudulent financial statements. The analysis in Section 2.1 showed that the calculated level of fraud was not necessarily an indication of falsification of the data by the enterprise. In certain cases, deviation of the financial data may be influenced by other factors such as loss of markets, change in credit conditions, etc. Controversial results may also be obtained in case of detection of fraud for one element of accounting only. For the reasons mentioned above and based on the analysis of the literature in Section 2.2, the fuzzy model was considered to be the most appropriate for the research.

The MATLAB™ software and fuzzy logic toolbox were used. Application of the fuzzy logic method under the Mamdani method was selected. The six input variables (the level of fraud of revenue, expenses, profit (loss), assets, equity, and liabilities) and one output variable were formed to determine the level of the financial statement fraud. Weights were set for the input and output variables. The following weights of the input variables were set: 0.00–0.25 – no fraud; 0.25–0.60 – possible fraud; 0.40–1.00 – fraud. The following weights of the output variables were set: 0.00–0.25–0.40 – no fraud; 0.40–0.50–0.60 – between possible fraud and no fraud; 0.60-0.70-1.00 – fraud. It was noticed that, in practice, the financial statements were unlikely to be completely fraudulent, and the value of the investigation would be 1 (total fraud). Fraud is usually committed in relation to actual enterprise data; therefore, it is impossible to actually have a financial statement containing total fraud (weight 1). In the real world, it is also unlikely to find an enterprise, all ratios of which would indicate that financial statements are absolutely fair (weight 0). However, depending on the data set used, it is possible that the sample enterprises will not have a weight of 0, as on average, all enterprises feature a certain level of fraud. The results may also be affected by sectors. This could apply to service enterprises, where the level of assets is low and the ratios related to assets may indicate fraud, when, in fact, it does not exist.

A solution rule base consisting of 90 IF-THEN rules was created (see Tab. 7). The rules were designed on the basis application of double entry in accounting: a fraud of profit and (or) equity may be affected by a fraud of revenue and expenses; liabilities and (or) expenses and

1. Selection of financial ratios and models

Financial ratios: DSRI; GMI; SGI; GP; SGAI; LEVI; ROCA; SFR; ATO; CATA; ITA; CTA	Financial models: Altman Z-score; logistic regression P-score

⇓

2. Financial ratios and models grouping by elements of accounting

Relationship with Revenue	Relationship with Expenses	Relationship with Profit	Relationship with Assets	Relationship with Equity	Relationship with Liabilities

⇓

3. Estimation of fraud in the financial statements of selected enterprises

Calculation of financial ratios and models	Encoding of financial ratios and model (0 – no fraud, 0.5 – possible fraud, 1 – fraud)	Average fraud of each elements of accounting group

⇓

4. Application of fuzzy logic

Design of Fuzzy IF-THEN rules, their weights and fuzzy model

Fig. 1: Research scheme for detection of fraudulent financial statements

Tab. 6: Examples of financial ratios and models for encoding thresholds

Ratios and models encoding	**0 – no fraud**	**0.5 – possible fraud**	**1 – fraud**
DSRI	DSRI < 1.00	1 < DSRI < 1.46	DSRI > 1.46
GMI	GMI < 1	1 < GMI < 1.19	GMI > 1.19
SGI	SGI < 1.1	1.1 < SGI < 1.6	GMI > 1.6
SGAI	SGAI < 1.001	1.001 < SGAI < 1.041	SGAI > 1.041
GP	GP > 0.362	0.086 < GP < 0.362	GP < 0.086
TATA	TATA < 0.016	0.016 < TATA < 0.031	TATA > 0.031
LEVI	LEVI < 1	1 < LEVI < 1.111	LEVI > 1.111
ROCA	ROCA > 0.299	0.057 < ROCA < 0.299	ROCA < 0.057
SFR	SFR > 0.586	0.453 < SFR < 0.586	SFR < 0.453
ATO	ATO < 1.54	1.54 < ATO < 2.243	ATO > 2.243
CATA	CATA < 0.473	0.473 < CATA < 0.667	CATA > 0.667
ITA	ITA < 0.19	0.19 < ITA < 0.321	ITA > 0.321
CTA	CTA < 0.035	0.035 < CTA < 0.091	CTA > 0.091
Z-score	Z > 2.6	1.1 < Z < 2.6	Z < 1.1
P-score	P < 0.4	0.4 < P < 0.5	P > 0.5

(or) equity may be affected by a fraud of assets; expenses and (or) assets may be affected by a fraud of liabilities; etc.

It should be noted that the number of rules drawn up may be determined by various factors such as the researcher's accounting experience, peculiarities of the national accounting framework, size of the enterprises, and other factors. The model developed by the authors of the present paper was applied to detection of fraudulent financial statements and interpretation of the results obtained in this research. The linguistic values (see Fig. 2) were used for determination of the fraudulence level.

Tab. 7: The fuzzy input and output values used in the authors' Mamdani model

Code: 1 – no fraud; 2 – possible fraud; 3 – fraud
Input: column 1 revenue; 2 – expenses; 3 – profit; 4 – assets; 5 – equity; 6 – liabilities
Output: column 7 – fraudulence level

R1: 1 1 1 1 1 1, 1	R19: 1 3 2 1 1 2, 2	R37: 3 1 3 1 3 1, 3	R55: 1 1 1 2 2 1, 2	R73: 1 1 1 3 2 1, 2
R2: 2 1 1 1 1 1, 1	R20: 2 1 3 2 1 1, 2	R38: 3 1 3 1 1 3, 3	R56: 1 1 1 2 3 1, 3	R74: 1 1 1 3 3 1, 3
R3: 1 2 1 1 1 1, 1	R21: 2 1 3 1 2 1, 2	R39: 1 3 3 3 1 1, 3	R57: 1 1 1 3 2 1, 3	R75: 1 1 1 1 2 2, 2
R4: 1 1 2 1 1 1, 1	R22: 2 1 3 1 1 2, 2	R40: 3 1 1 3 1 1, 3	R58: 1 1 1 3 1 3, 3	R76: 1 1 1 1 2 3, 2
R5: 1 1 1 2 1 1, 1	R23: 1 2 3 2 1 1, 2	R41: 3 3 3 3 3 3, 3	R59: 1 1 1 2 1 3, 3	R77: 1 1 1 1 3 2, 2
R6: 1 1 1 1 2 1, 1	R24: 1 2 3 1 2 1, 2	R42: 3 3 3 3 3 2, 3	R60: 1 1 1 3 1 2, 3	R78: 1 1 1 1 3 3, 3
R7: 1 1 1 1 1 2, 1	R25: 1 2 3 1 1 2, 2	R43: 3 3 3 3 3 1, 3	R61: 1 1 1 2 1 2, 2	R79: 2 2 2 2 1 1, 2
R8: 2 1 2 2 1 1, 2	R26: 2 1 2 3 1 1, 2	R44: 3 3 3 2 3 3, 3	R62: 1 2 1 2 1 1, 2	R80: 2 2 2 3 1 1, 2
R9: 2 1 2 1 2 1, 2	R27: 2 1 2 1 3 1, 2	R45: 3 3 3 2 2 3, 3	R63: 1 3 1 2 1 1, 3	R81: 2 3 2 3 1 1, 3
R10: 2 1 2 1 1 2, 2	R28: 2 1 2 1 1 3, 2	R46: 3 3 3 2 1 3, 3	R64: 1 2 1 3 1 1, 3	R82: 2 3 3 3 1 1, 3
R11: 1 2 2 2 1 1, 2	R29: 1 2 2 3 1 1, 2	R47: 3 3 2 1 1 2, 3	R65: 1 3 1 3 1 1, 3	R83: 3 3 3 3 1 1, 3
R12: 1 2 2 1 2 1, 2	R30: 1 2 2 1 3 1, 2	R48: 3 3 3 1 1 2, 3	R66: 2 1 1 2 1 1, 2	R84: 2 2 2 1 2 1, 2
R13: 1 2 2 1 1 2, 2	R31: 1 2 2 1 1 3, 2	R49: 3 3 3 1 1 3, 3	R67: 2 1 1 3 1 1, 3	R85: 3 2 2 1 2 1, 2
R14: 3 1 2 2 1 1, 2	R32: 3 1 2 1 3 1, 3	R50: 1 1 1 2 2 2, 2	R68: 3 1 1 2 1 1, 3	R86: 3 3 2 1 2 1, 3
R15: 3 1 2 1 2 1, 2	R33: 3 1 2 1 1 3, 3	R51: 1 1 1 3 2 2, 2	R69: 1 3 3 1 3 1, 3	R87: 3 3 3 1 2 1, 3
R16: 3 1 2 1 1 2, 2	R34: 1 3 2 3 1 1, 3	R52: 1 1 1 3 3 2, 3	R70: 1 3 3 1 1 3, 3	R88: 3 3 3 1 3 1, 3
R17: 1 3 2 2 1 1, 2	R35: 1 3 2 1 3 1, 3	R53: 1 1 1 3 3 3, 3	R71: 1 1 1 2 2 1, 2	R89: 2 2 2 1 1 2, 2
R18: 1 3 2 1 2 1, 2	R36: 1 3 2 1 1 3, 3	R54: 1 1 1 3 3 1, 3	R72: 1 1 1 2 3 1, 2	R90: 3 2 2 1 1 2, 2

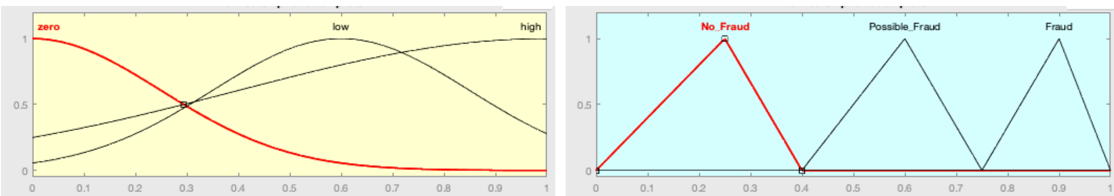

Fig. 2: The fuzzy input and output values used in the authors' Mamdani model

4 RESULTS

4.1 Methods of Analysis and Research Results

Three methods of data analysis were used: fuzzy reasoning, conventional statistics, and the generalized mean (Dyckhoff and Pedrycz, 1984). The input data were collected as mentioned above. Meanwhile, the corresponding outputs for the variable risk of fraudulence, namely, the fraudulence level, were first generated by applying the authors' fuzzy rule-based Mamdani model that included 90 rules altogether (Mamdani and Assilian, 1975). The rules had the following form:

If Revenue is _ and Expenses are _
and Profit is _ and Assets are _
and Equity is _ and Liability is _,
then Fraudulence level is _. (1)

Linguistic variable values zero, low and high for the Fraudulence level were used. For example:

If Revenue is high and Expenses are low
and Profit low and Assets are zero
and Equity is zero and Liability is zero,
then Fraudulence level is high. (2)

The fuzzy model was applied to the empirical input data during calculation of the fraudulence levels. The augmented data set was used in the investigations below (Fig. 3, Tab. 7).

The mean was noticed to be approximately 0.64. The range and standard deviation were quite small in the case of the fraudulence levels (see Tab. 8).

Various intercorrelations prevailed between the variables due to their expected real in-

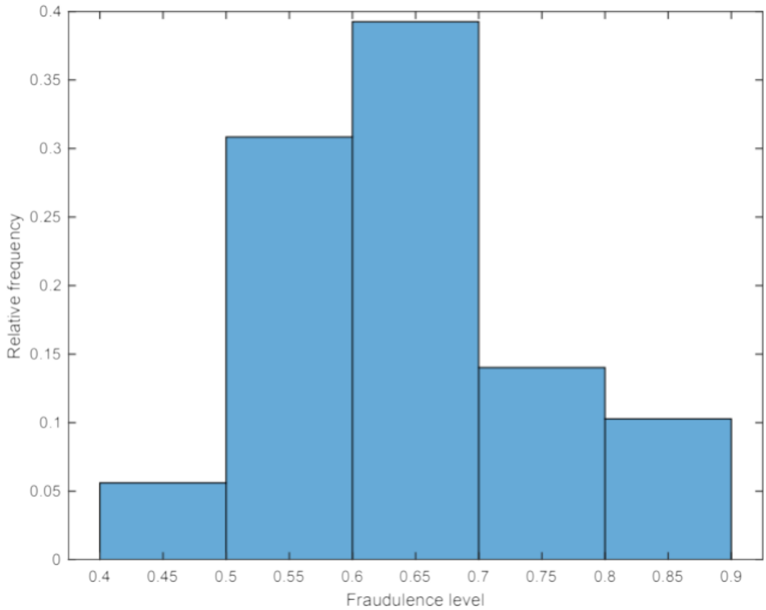

Fig. 3: Fuzzy Mamdani model outputs for fraudulence level with the research data

Tab. 8: Descriptive statistics of the research data ($N = 107$)

	Minimum	**Maximum**	**Mean**	**Std. deviation**
Revenue	0.0000	0.6875	0.349883	0.1575675
Expenses	0.0000	1.0000	0.460300	0.3636600
Profit	0.0000	1.0000	0.510125	0.2155846
Assets	0.0789	0.7632	0.374570	0.1571723
Equity	0.0000	1.0000	0.490654	0.2198513
Liabilities	0.0000	1.0000	0.273364	0.2445640
Fraudulence level	0.4355	0.8811	0.640106	0.1073657

Tab. 9: Intercorrelations between the variables ($N = 107$)

		Revenue	**Expenses**	**Profit**	**Assets**	**Equity**	**Liabilities**	**Fraudulence level**
Revenue	Pearson Correlation	–						
Expenses	Pearson Correlation	0.536	–					
	Sig. (2-tailed)	< 0.011						
Profit	Pearson Correlation	0.061	0.090	–				
	Sig. (2-tailed)	0.532	0.354					
Assets	Pearson Correlation	0.682	0.105	0.308	–			
	Sig. (2-tailed)	< 0.001	0.280	0.001				
Equity	Pearson Correlation	0.201	0.132	0.900	0.519	–		
	Sig. (2-tailed)	0.030	0.176	< 0.001	< 0.001			
Liabilities	Pearson Correlation	0.525	0.059	0.516	0.774	0.719	–	
	Sig. (2-tailed)	< 0.001	0.549	< 0.001	< 0.001	< 0.001		
Fraudulence level	Pearson Correlation	0.523	0.398	0.797	0.623	0.865	0.753	–
	Sig. (2-tailed)	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	

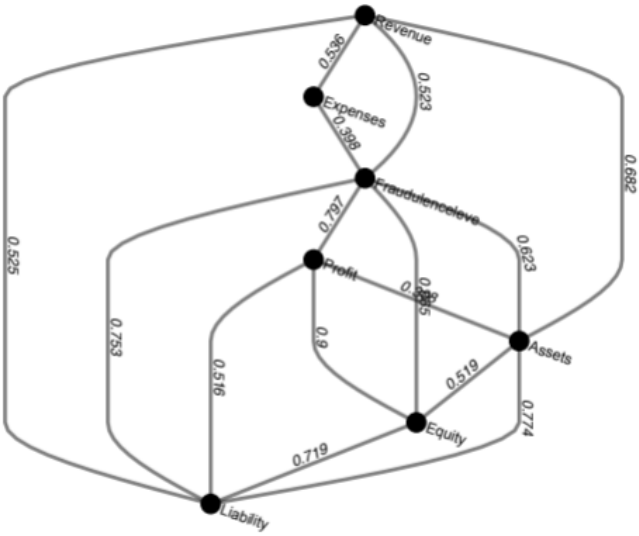

Fig. 4: Depiction of intercorrelations between the research variables

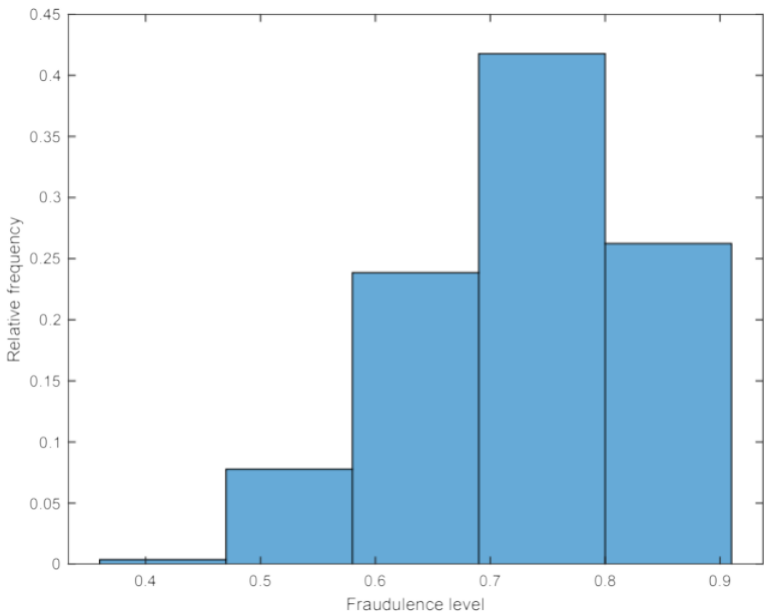

Fig. 5: Fuzzy Mamdani model outputs for the fraudulence level with 200,000 random input vectors

terconnections. It was nevertheless decided to keep all the original variables in the model design (Fig. 4, Tab. 9). For example, it is a well-known fact in accounting that assets and equity will eventually have a strong connection. Generally, everything seems to depend on everything else in this problem setting. Hence, to study the real essence of fraudulence, possible interactions between the variables also had to be considered.

For better justification of the fuzzy model proposed by the authors, testing simulations were performed with 200,000 random input vectors, the component values of which ranged from 0 to 1. The outputs obtained ranged from 0.395 to 0.882 and the expected value thereof

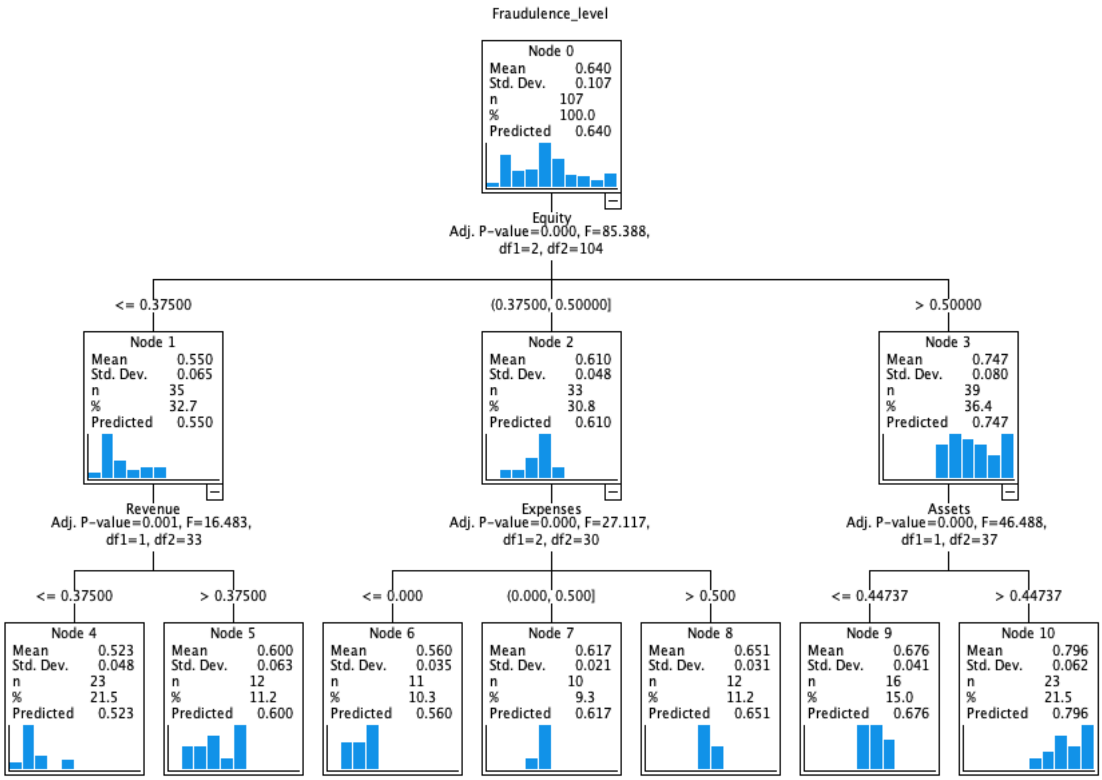

Fig. 6: Decision tree for fraudulence levels with the research data

was 0.726 with the standard deviation of 0.095 (Fig. 5). The quartile values thereof were 0.672, 0.723 (median) and 0.805, respectively. The fuzzy model proposed by the authors thus seems to have yielded a somewhat negatively skewed normal distribution for the fraudulence level values.

Since the fuzzy model proposed by the authors was based on the data of the selected Lithuanian enterprises, this may explain why the fraudulence level results were not higher than 0.9. In fact, very high fraudulence levels would have indicated that the given financial statements were false. This situation seems counterfactual in the real world because the statements are issued according to the financial accounting data of the enterprises. Hence, very high fraudulence level values in the authors' model seem to suggest that the input data provided are not based on the actual accounting data.

On the other hand, the fraudulence values may be affected by human factors such as accounting errors. These factors may be the possible causes behind the failure of the proposed fuzzy model to yield fraudulence level values less than 0.4. It is therefore assumed that the proposed fuzzy model seems to be appropriate for analysis of the fraudulence levels in the above-mentioned context.

The decision tree analysis under the exhaustive CHAID method was applied by using SPSSTM for the examination of how the discrimination of the empirical data was performed with respect to the fraudulence levels (Fig. 6). In the model proposed, which only allowed three classes of the variables, the risk of erroneous predictions was only 0.2 per cent. Hence, in the

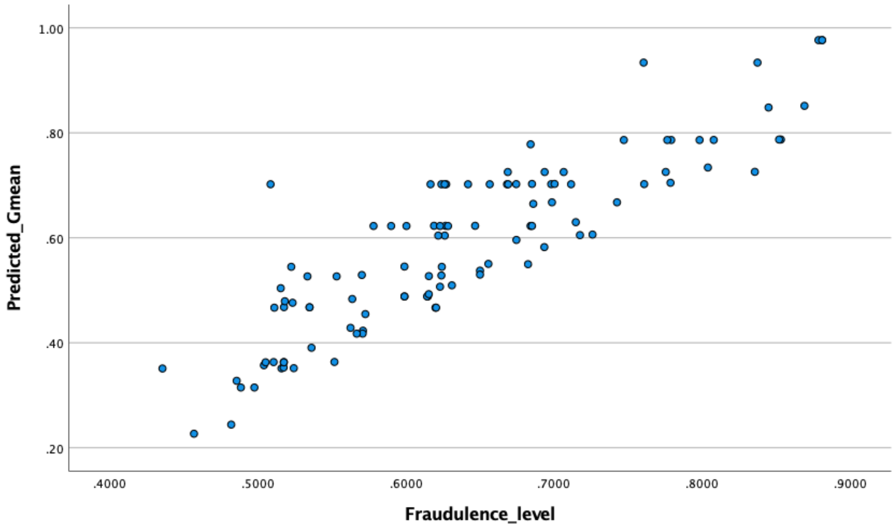

Fig. 7: Predicted vs. observed values in the generalized mean model

decision model, these paths yielded the highest fraudulence levels:

If Equity ≤ 0.375 and Revenue > 0.375, mean Fraudulence level $= 0.6$ (3)

If $0.375 <$ Equity ≤ 0.5 and Expenses > 0.5, mean Fraudulence level $= 0.651$ (4)

If Equity > 0.5 and Assets > 0.447, mean Fraudulence level $= 0.796$ (5)

On the other hand, the path for the lowest fraudulence levels was:

If Equity < 0.375 and Revenue < 0.375, mean Fraudulence level $= 0.523$ (6)

The decision tree proposed seems to confirm the basic principle of accounting, namely, Assets = Liabilities + Equity. Hence, when Equity increases, Assets increase as well. This connection may be explained by the fact in accounting that a fraudulent change in the value of its assets by an enterprise will also lead to a change in the amount of equity.

Finally, the generalized mean was applied as an aggregation operator because this method may reveal how the input variables, X, were weighted and compensated with respect to the output variable, Y (Yager, 1988). In this sense, this operation represents descriptive multi-criteria decision-making assessment. The following formula may thus be applied:

$$Y = \left(\sum_i W_i \times X_i^p \right)^{1/p}, \tag{7}$$

$$0 \leq X_i \leq 1, \quad \sum_i W_i = 1, \quad p \in \mathbb{R},$$

in which weights, W, and degree of compensation, p, are optimized.

Hence, for example, the following interpretations may be made:

If $p \to -\infty$, $Y = \min(X_i)$ (8)
If $p = -1$, Y = harmonic mean of X_i
If $p = 0$, Y = geometric mean of X_i
If $p = 1$, Y = mean of X_i
If $p = 2$, Y = quadratic mean of X_i
If $p \to \infty$, $Y = \max(X_i)$

In the case considered, the genetic algorithm of MATLAB™ was applied first, and then the parameters were fine-tuned with the Levenberg-Marquart algorithm. These methods were used several times to verify the stability of the parameter values. It was noticed that the generalized mean models yielded RMSEs < 0.01

and variables Assets and Equity were the most important factors with their approximate weight values of 0.3. Meanwhile, the weights of other variables were close to zero. The degree of compensation, p, was greater than 20, and the highest input variable values thus seemed to have the strongest effect on the fraudulence level values (Fig. 7).

The above outcomes, namely, the highest input vector values, which had the strongest effect on the fraudulence levels, were also obtained with the OWA compensation operator that was applied in order to double-check the nature of compensation (Yager, 1988). Under the OWA method, both the weights, W, and the input vector values, X, were first ordered, and then the weighted mean of inputs, Y, were calculated:

$$Y = \left(\sum_i W_i \times X_i \right), \tag{9}$$

$$0 \leq W_i \leq 1, \quad \sum_i W_i = 1.$$

Hence, the highest weights were used for the highest input values and vice versa. The model yielded RMSE $= 0.01$ with the above optimization methods and the parameters, W. The highest inputs generated the approximate weight values of 0.45 in the computer simulations performed.

A checking data set, which was not used in the fuzzy model construction, was also collected for more thorough examination of the model proposed. However, since only the input values were available, their output values, the fraudulence levels, were calculated again with the proposed fuzzy Mamdani model (Tab. 10). It was therefore impossible to use the actual checking data for cross-validation.

In this context, the highest input variable values were also noticed to have the strongest effect on the fraudulence level estimation when the generalized mean and OWA methods were applied to the optimization simulations performed.

As mentioned above, the rules used in the fuzzy model proposed were based on the principle of double entry. Therefore, if two elements of accounting indicated a higher level of fraudulence, fraudulent financial statements were detected. Case 6 in the checking data (Tab. 10) may confirm this idea. If the profit had been fraudulent (0.67) under the double entry, the change would have been visible in equity (0.75). When similar relationship was detected, it could be concluded that the financial statements would be fraudulent (0.67). When using the linguistic variables (Huda et al., 2015), this meant the real fraud (weight 0.61–0.75). Case 3 in the checking data (Tab. 10), in turn, showed different situation at the enterprise. Fraud in expenses could be seen (weight 0.41–0.60), but other accounting elements indicated (Huda et al., 2015) that the fraud did not exist (weight 0.01–0.25). Based on the linguistic variables, this cannot be claimed to be a real fraud. In case of checking data number 3, it could potentially be claimed to be a fraud, because the changes in profit were not reflected in another elements of accounting under the double entry.

The limitation of our research is that we did not have the possibilities to check the financial data of those enterprises which were punished by court for financial crimes. The object of our research was micro and small enterprises that are not public and thus in court cases for financial fraud the names of enterprises and their owners as well as managers are not disclosed in publicly due to the data protection regulation.

Summarizing the results of our research, it could be stated that, in the case of the Lithuanian micro and small enterprises, the developed fuzzy model based on the selected financial ratios and models can be used for determination of the level of fraud. Based on the selected enterprises sample ($N = 107$) in the present research, the level of fraudulence in different enterprises ranged from 0.4 to 0.9. Similar results were obtained using the checking data. In the case of checking data ($N = 13$), the level of fraud ranged approximately from 0.4 to 0.7. Based on the linguistic variables, a real fraud can be considered to exist when then fraudulence levels rise to 0.61. The developed fuzzy model for detection of fraudulent financial

Tab. 10: Checking data inputs and the predicted fraudulence levels thereof

N	**Revenue**	**Expenses**	**Profit**	**Assets**	**Equity**	**Liabilities**	**Fraudulence**
1	0.19	0.00	0.58	0.42	0.44	0.13	0.52
2	0.44	0.50	0.33	0.55	0.31	0.38	0.62
3	0.25	0.50	0.08	0.13	0.06	0.13	0.44
4	0.19	0.00	0.08	0.24	0.31	0.25	0.50
5	0.50	0.50	0.08	0.37	0.06	0.19	0.58
6	0.13	0.00	0.67	0.29	0.75	0.25	0.67
7	0.31	0.00	0.08	0.39	0.06	0.25	0.52
8	0.19	0.00	0.08	0.34	0.06	0.13	0.47
9	0.19	0.25	0.67	0.18	0.50	0.00	0.61
10	0.13	0.00	0.67	0.26	0.50	0.00	0.59
11	0.38	0.00	0.58	0.42	0.56	0.31	0.61
12	0.44	0.50	0.67	0.45	0.63	0.38	0.72
13	0.13	0.00	0.33	0.37	0.31	0.06	0.51

statements enables determination of not only the level of fraud in the financial statements, but also estimation of the level of fraud in each individual element of accounting. This provided additional information and the possibility to determine what manipulations might have taken place at the enterprise in order to prepare the fraudulent financial statements.

5 DISCUSSION

The study presented in the paper pursued the second objective and focused on the design of a model that would enable detection of the fraudulence level of a financial statement. The advantage of the model proposed is that the model is based on the financial ratios and models which are available for the financial statements of enterprises. In terms of practical application, the model facilitates the detection of fraudulent financial statements as it eliminates the need for additional non-financial indicators. Certain researchers have proposed the Beneish M-score (Mamo and Shehu, 2015; Mavengere, 2015; Aghghaleh et al., 2016; MacCarthy, 2017; Bhavani and Amponsah, 2017; Teherinia et al., 2019; Halilbegovic et al., 2020) and Dechow F-score (Aghghaleh et al., 2016) financial models that are characterised by fairly high reliability. In the case of the present study, however, these models could not be applied to micro and small enterprises due to the abridged financial statements used by them. The authors therefore explored other options and proposed a fuzzy model for detection of fraudulent financial statements.

The fuzzy model proposed is intended to be used as a tool for managerial decision-making at enterprises. Meanwhile, the models proposed by other researchers are based on the listed enterprises and focus more on support to the investment process (Nawrocki, 2018). The fuzzy model proposed here may help the owners and managers of micro and small enterprises detect fraud of business partners and reduce the business risk. This study may be helpful to the regulators for identification of the fraudulence level in the financial statements. The model presented may be used as a prevention tool to reduce unfair business practice and to help fair businesses compete on the market.

With the larger set of variables in the fuzzy model proposed, the model is believed to ensure more reliable results. Twelve financial ratios and two financial models were combined and grouped by elements of accounting. Compared to other studies, larger number of indicators was selected, although there were certain limitations related to the abridged version of financial statements. Meanwhile, different numbers of financial variables were found in other studies. In these studies (Čičak and Vašiček, 2019; Rostamy-Malkhalifed et al., 2021), six financial variables were used, in study (Chen et al., 2014) – seven financial variables, and in (Omar et al., 2017) – ten financial variables. Certain studies were different in that they focused on the tax issue and included the taxation-related variables (Čičak and Vašiček, 2019). The study presented in the paper did not investigate the compatibility of tax rules with accounting rules and its effects on the fraud in financial statements. This could be a new direction in the future research.

Only a few studies used a larger number of financial variables: fourteen in (Maltoudoglou et al., 2015) and fifteen in (Nawrocki, 2018). The studies with larger number of financial ratios often applied the fuzzy logic models (Maltoudoglou et al., 2015; Nawrocki, 2018), but the applied financial ratios differed from the ones in the present research as the former used the listed enterprises. The main difference of the fuzzy model proposed here and the ones proposed in the previous research was the number of fuzzy rules: 90 rules used in the present study versus 27 fuzzy rules in (Nawrocki, 2018) and 81 rules in (Čičak and Vašiček, 2019).

In summary, the proposed model has the following advantages. First, the model is based on larger number of financial ratios and models which are applicable in practice. Second, it can be applied to micro and small enterprises both as a decision-making tool and as a regulatory prevention tool for fair activity of the enterprises. Third, the fuzzy model is more reliable due to the large number of fuzzy rules, thus enabling it to identify correctly small deviations in fraudulent financial statements. Hence, the large number of fuzzy rules will avoid better the misinterpretations stemming from the grey area. The results suggest that the proposed model can be improved with a larger sample size and using neural networks in the future research.

6 CONCLUSIONS

In this paper, a fuzzy model for detection of fraudulent financial statements was proposed based on fourteen financial indicators combined into six input variables according to the relationships with the elements of accounting (revenue, expenses, profit, assets, equity, liabilities). Such a combination of variables provides insights into possible cases of fraud in relation to these elements of accounting, and not just assessment of fraud in financial statements in general. The uniqueness of the present research lies in that it focuses on micro and small enterprises rather than on the listed enterprises (available as public data) used by the previous study. A major limitation of the research presented was that it was impossible to include non-financial indicators (number of employees, business sector, etc.) in the model as depersonalized information about the enterprises was used in line with the General Data Protection Regulation. For this reason, we were also unable to test the model with the actual confirmed fraudulent financial statements. In the future research, we will look for possibilities to obtain depersonalized cases of financial crime in a micro and small enterprises to improve our proposed model.

A larger sample of data of micro and small enterprises could not be collected for the research; therefore, the neural network approach was not applied. Based on the scientific literature, fuzzy logic was chosen. It enabled the authors to identify a model for detection of fraudulent financial statements. The results of the designed fuzzy logic model were tested using the decision tree. Although this test have shown that the results were close to each other. Although the model was designed for micro and small enterprises, it could also be applied to medium-sized and large enterprises. The model cannot be applied to micro enterprises if they choose to prepare profit (loss) statements by nature. In practice, the model proposed has been designed to help small businesses reduce the risk, but it may also be used by public authorities as a tool for achieving greater business transparency. The proposed fuzzy model for detection of fraudulent financial statements can be adapted to other countries to take into account the national differences in financial statements. Future work include exploration of a new fuzzy model for detection of financial statement fraudulence taking in account the type of enterprise activity (services, trade, manufacturing) and industry sector. These aspects may influence the selection of financial indicators for design of the fuzzy model, while differences in the results may be influenced by the mentioned aspects (activity and sector).

7 ACKNOWLEDGEMENT

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